ISSN 0005-1179 (print), ISSN 1608-3032 (online), Automation and Remote Control, 2025, Vol. 86, No. 9, pp. 831–844.


SPECIAL ISSUE

# Robust Control Design Based on Noisy Current Data

**R. S. Biryukov**[*,a] **and M. M. Kogan**[**,b]

[*]*Lobachevsky State University of Nizhny Novgorod, Nizhny Novgorod, Russia*
[**]*Sirius University of Science and Technology, Sochi, Russia*
*e-mail:* [a]*biryukovrs@gmail.com*, [b]*mkogan@nngasu.ru*

Received March 3, 2025
Revised May 20, 2025
Accepted June 27, 2025

**Abstract**—This paper presents a new approach to designing robust controllers for an uncertain dynamic system using the generalized $H_\infty$ norm as a criterion. The controller parameters are tuned during the real-time operation of the system as current measurements of the state and control, obtained with error, become available. The numerical simulation results of the active vibration suppression of buildings under seismic loads illustrate the effectiveness of the approach proposed.

*Keywords*: current data, robust control, generalized $H_\infty$ norm, dual systems, linear matrix inequalities (LMIs)

**DOI:** 10.31857/S0005117925090038

## 1. INTRODUCTION

In this paper, we develop a new approach to designing optimal controllers for linear dynamic systems with unknown mathematical models, unmeasurable disturbances, and uncertain initial conditions, utilizing a priori information and current data in an online mode. As is known, one of the most common robust control methods is based on the minimax approach: design a controller ensuring the best estimate of an optimized functional over the entire set of uncertain parameter values, determined based on a priori information (for example, see [1, 2]). Despite all its advantages, this approach has a significant drawback: the guaranteed upper bound of the functional is quite rough due to the large size of the uncertainty set, and in the absence of accurate a priori information, the uncertainty set can become so large that a robust controller ceases to exist.

Recently, there has been significant interest in the development of control methods using data obtained preliminarily during an experiment, often referred to as "data-driven control" [3, 4]. Initially, these methods were based on the fundamental result of [5], showing that a single trajectory obtained under the so-called persistent excitation condition can be used for the complete characterization of a linear time-invariant dynamic system without disturbances. According to [6], the parameterization of a closed-loop control system based on experimental data allows designing controllers using linear matrix inequalities (LMIs). As established in [7], for designing controllers from experimental data, it suffices to satisfy the less restrictive condition of data informativity with respect to the property under study. Subsequently, these methods were extended to systems with disturbances [8–11] and linear time-varying systems [12]; a stabilizing controller based on measurements obtained with an error was designed in [13].

The mathematical modeling results presented in the above works demonstrate that even for relatively small disturbance amplitudes, the guaranteed estimate of the control performance index turns out to be significantly overvalued. This happens because, as the noise amplitude increases, the set of systems consistent with the experimental data expands significantly. In most of these works, experimental data must also satisfy certain requirements: the matrix composed of state and control measurements along the system trajectory must be of full row rank. To satisfy this rank condition, the input signals in the experiments must provide persistent excitation in the system, necessary for the identifiability of the unknown parameters. Although persistent excitation in the system is not formally required to ensure data informativity, the uncertainty set can become unbounded even for small noise amplitudes if the rank condition fails for the experimental data.

These circumstances suggest the idea of designing a controller based on both a priori information and experimental data. In [14–20], the parameters of state-feedback controllers for linear time-invariant systems on an infinite horizon, time-varying systems on a finite horizon, as well as some classes of nonlinear systems, were found with the joint use of a priori information and experimental data by solving a minimax problem on the set of unknown parameters consistent with the a priori information and experimental data. Such controllers, obtained by solving LMIs, provide a certainly smaller value of the guaranteed functional estimate than, first, classical robust controllers and, second, a controller designed only from experimental data.

During the experimental stage, which precedes the controller design, input signals are chosen based on the informativity requirements for the output data. However, this stage is not always feasible due to, e.g., the instability of the system. Furthermore, the parameters of some control systems depend on the state of the environment, e.g., temperature or pressure. In these cases, control designed from preliminary stage data may be just partially suitable under other conditions. Instead, it would be desirable to retune the controller parameters at each step of the real-time system operation as new current data arrive, based on solving a minimax problem for the set of models consistent with the information available by that time. Since this set narrows (or, at worst, does not increase) at each step, the feedback controllers determined in this way will provide monotonically decreasing or non-increasing guaranteed values of the functional for the unknown system. The parameter tuning procedure stops when the guaranteed functional value becomes less than a desired one. This approach is developed below. Note that the informativity conditions are not required, and the significant question about the necessary number of measurements is eliminated as well. An illustrative example of active vibration suppression of buildings under seismic loads demonstrates the effectiveness of the controller design.

## 2. PROBLEM STATEMENT

Consider an uncertain system of the form

$$\begin{aligned} x_{t+1} &= A_{real}x_t + B_{real}u_t + B_w w_t, \quad x(0) = x_0, \\ z_t &= Cx_t + Du_t, \quad t = 0, 1, \dots, \end{aligned} \tag{2.1}$$

with the following notation: $x_t \in \mathrm{R}^{n_x}$ is the state vector, $u_t \in \mathrm{R}^{n_u}$ is the control vector (input), $w_t \in \mathrm{R}^{n_w}$ is a disturbance, and $z_t \in \mathrm{R}^{n_z}$ is the performance (controlled) output. By assumption, the initial state $x_0$ can be arbitrary, the disturbance $\{w_t, t \geqslant 0\}$ has a bounded $l_2$ norm, i.e., $\|w\| = (\sum_{t=0}^{\infty} |w_t|^2)^{1/2} < \infty$, where $|\cdot|$ stands for the Euclidean norm of an appropriate vector, and the state is available for control in the feedback loop. The system matrices, collected in the matrix $\Omega_{real} = (A_{real}\ B_{real})$, are unknown, but the pair $(A_{real}, B_{real})$ is supposed to be stabilizable; finally, the matrix $B_w$ (in particular, an identity matrix) is known.

Since the system matrices are unknown, standard optimal control methods turn out to be inapplicable. In this regard, we will involve a priori information, as is customary in robust control, and current data, characteristic of adaptive control, for control design. Assume that in the system, the disturbance $w_t$ has a bounded Euclidean norm and the state and control are measured with some bounded noise:

$$\widehat{x}_t = x_t + \xi_t, \quad \widehat{u}_t = u_t + \eta_t, \tag{2.2}$$

where $\widehat{x}_t$ and $\widehat{u}_t$ are the state and control measurements, respectively, at a time instant $t$, and

$$|\xi_t| \leqslant \varepsilon_\xi, \quad |\eta_t| \leqslant \varepsilon_\eta, \quad |w_t| \leqslant \varepsilon_w \quad \forall\, t \geqslant 0. \tag{2.3}$$

Note that the state $\widehat{x}_t$ and control $\widehat{u}_t$ measurements, which will be included in the current data, contain the specified bounded errors, and the state $x_t$ is available for control.

Generally, it is required to design linear state-feedback controllers with tunable parameters based on a priori information and current data during the real-time operation of the system in order to reach, after some time, steady-state values that minimize the effect of the initial conditions and exogenous disturbance on the performance output of the closed-loop time-invariant system.

## 3. CHARACTERIZATION OF THE NORM IN DUAL SYSTEM TERMS

Before proceeding to the problem solution, let us recall the main results regarding the generalized $H_\infty$ norm from the input $w_t$ to output $z_t$ of a given closed-loop system with a controller $u_t = \Theta x_t$, described by the equations

$$\begin{aligned} x_{t+1} &= (A + B\Theta)x_t + B_w w_t, \\ z_t &= (C + D\Theta)x_t. \end{aligned} \tag{3.1}$$

The generalized $H_\infty$ norm characterizes the effect of initial conditions and an exogenous disturbance on the $l_2$ norm of the performance output (i.e., on the transients) and is defined as

$$\|H\|_{g\infty}(\Omega, \Theta) = \sup_{\{x_0,\, w_t \in l_2\}} \frac{\|z\|}{\left(x_0^{\mathrm{T}} R^{-1} x_0 + \|w\|^2\right)^{1/2}}, \tag{3.2}$$

where $\Omega = (A \ B)$ and $R = R^{\mathrm{T}} \succ 0$ is a weight matrix measuring the comparative significance of the uncertainty in the initial conditions and exogenous disturbance for the performance output; $S \succ T$ ($S \succeq T$) means that the matrix $S - T$ is positive definite (positive semidefinite, respectively). In this definition, by assumption, the denominator on the right-hand side of (3.2) does not vanish and

$$H(s) = (C + D\Theta)[sI - (A + B\Theta)]^{-1} B_w$$

is the transfer matrix of the closed-loop system relating the disturbance to the performance output. In particular, the generalized $H_\infty$ norm of the system with zero initial conditions coincides with the standard $H_\infty$ norm; in the absence of an exogenous disturbance, it characterizes the maximum value of the quadratic functional of the performance output for an initial state belonging to the ellipsoid $x^{\mathrm{T}} R^{-1} x \leqslant 1$. The following results on the computation of these norms will be applied below.

**Lemma 3.1** [21]. *The generalized $H_\infty$ norm of system* (3.1) *satisfies the condition $\|H\|_{g\infty} < \gamma$ if and only if there exists a positive definite quadratic form $V(x) = x^{\mathrm{T}} Y x$, $Y \prec \gamma^2 R^{-1}$ such that*

$$\Delta V(x_t) + |z_t|^2 - \gamma^2 |w_t|^2 < 0, \tag{3.3}$$

*for all nonzero $x_t$ and $w_t$, where $\Delta V(x_t) = V(x_{t+1}) - V(x_t)$ denotes the increment of the function $V(x)$ along the trajectory of the corresponding system.*

**Lemma 3.2** [17]. *The generalized $H_\infty$ norm of system* (3.1) *satisfies the condition $\|H\|_{g\infty} < \gamma$ if and only if there exists a positive definite quadratic form $V_d(x) = x^{\mathrm{T}} P x$, $P \succ R$, whose increment along the trajectory of the dual system*

$$\begin{aligned} x_{t+1}^{(d)} &= (A + B\Theta)^{\mathrm{T}} x_t^{(d)} + (C + D\Theta)^{\mathrm{T}} w_t^{(d)}, \\ z_t^{(d)} &= B_w^{\mathrm{T}} x_t^{(d)} \end{aligned} \tag{3.4}$$

*is such that*

$$\Delta V_d(x_t^{(d)}) + |z_t^{(d)}|^2 - \gamma^2 |w_t^{(d)}|^2 < 0 \tag{3.5}$$

*for all nonzero* $x_t^{(d)}$ *and* $w_t^{(d)}$.

*Remark 3.1.* For the generalized $H_\infty$ norm, the matrices of the quadratic forms $V(x) = x^{\mathrm{T}} Y x$ and $V_d(x^{(d)}) = x^{(d)\mathrm{T}} P x^{(d)}$ of the primal and dual systems are related by $P = \gamma^2 Y^{-1}$.

*Remark 3.2.* Under the conditions of each of the lemmas above, the corresponding systems are asymptotically stable in the absence of disturbances.

## 4. A PRIORI INFORMATION AND CURRENT DATA

Following conventional robust control methods, let there be a priori information that the real matrix $\Omega_{real} = (A_{real}\ B_{real})$ lies strictly inside the domain defined by the inequality

$$(\Omega - \Omega_*)(\Omega - \Omega_*)^{\mathrm{T}} \preceq \rho^2 I, \quad \Omega_* = (A_*\ B_*)\,, \tag{4.1}$$

where $\Omega_*$ and $\rho$ characterize the nominal system and the size of the uncertainty domain, respectively. We write inequality (4.1) as

$$F_a(\Omega) := (\Omega \quad I)\, \Psi_a\, (\Omega \quad I)^{\mathrm{T}} \preceq 0, \tag{4.2}$$

where

$$\Psi_a = \left( \begin{array}{c|c} I & \star \\ \hline -\Omega_* & \Omega_* \Omega_*^{\mathrm{T}} - \rho^2 I \end{array} \right) \tag{4.3}$$

and $\star$ replaces the corresponding block of the symmetric matrix. The set $\Delta^{(a)} := \{\Omega \in \mathrm{R}^{n_x \times (n_x + n_u)} : F_a(\Omega) \preceq 0\}$ will be called the set of all matrices consistent with the a priori information. By assumption, $\Omega_{real} \in \Delta^{(a)}$.

Due to (2.1) and (2.2), the current data obtained at the time instant $t+1$ satisfy the equation

$$\widehat{x}_{t+1} = \Omega_{real} \widehat{\varphi}_t + (I\ \Omega_{real}\ B_w) \begin{pmatrix} \xi_{t+1} \\ \zeta_t \\ w_t \end{pmatrix}, \tag{4.4}$$

where

$$\widehat{\varphi}_t := \begin{pmatrix} \widehat{x}_t \\ \widehat{u}_t \end{pmatrix}, \quad \zeta_t =: - \begin{pmatrix} \xi_t \\ \eta_t \end{pmatrix}.$$

The set of all matrices $\Omega$ satisfying equation (4.4) for some disturbance and noises under the constraints (2.3) will be called the set of all matrices consistent with the data $\widehat{x}_t, \widehat{x}_{t+1}$, and $\widehat{u}_t$, and denoted by $\Delta_t^{(p)}$. Clearly, $\Omega_{real} \in \Delta_t^{(p)}$, and the set $\Delta_t^{(p)}$ corresponds to all systems that could have generated these data. This set is characterized as follows.

**Lemma 4.1.** *All matrices* $\Omega \in \Delta_t^{(p)}$ *consistent with the data* $\widehat{x}_t, \widehat{x}_{t+1}$, *and* $\widehat{u}_t$ *satisfy the inequality*

$$(\Omega \quad I) \begin{pmatrix} \widehat{\varphi}_t \widehat{\varphi}_t^{\mathrm{T}} - d^2 I & \star \\ -\widehat{x}_{t+1} \widehat{\varphi}_t^{\mathrm{T}} & \widehat{x}_{t+1} \widehat{x}_{t+1}^{\mathrm{T}} - (I + B_w B_w^{\mathrm{T}}) d^2 \end{pmatrix} (\Omega \quad I)^{\mathrm{T}} \preceq 0, \tag{4.5}$$

*where* $d^2 = 2\varepsilon_\xi^2 + \varepsilon_\eta^2 + \varepsilon_w^2$.

The proof of this and all subsequent results is given in the Appendix. Let some measurements of the states $\widehat{x}_0, \dots, \widehat{x}_k$ and previously chosen controls $\widehat{u}_0, \dots, \widehat{u}_{k-1}$ of system (2.1) under some unknown disturbances $w_0, \dots, w_{k-1}$ be obtained by a fixed time instant $k$. The matrices $\Omega$ consistent with all the data $\widehat{x}_0, \dots, \widehat{x}_k$ and $\widehat{u}_0, \dots, \widehat{u}_{k-1}$ obtained by a time instant $k \geqslant 1$ belong to the set $\Delta^{(p)}_{[0,\,k-1]} = \bigcap_{t=0}^{k-1} \Delta^{(p)}_t$ and are defined by inequalities (4.5) for $t = 0, \dots, k-1$. We denote by $\boldsymbol{\Delta}_0 = \Delta^{(a)}$ and $\boldsymbol{\Delta}_k = \Delta^{(a)} \bigcap \Delta^{(p)}_{[0,\,k-1]}$, $k \geqslant 1$, the set of all matrices $\Omega$ consistent with the a priori information and the data $\widehat{x}_0, \dots, \widehat{x}_k$ and $\widehat{u}_0, \dots, \widehat{u}_{k-1}$ obtained by a time instant $k$ for $k \geqslant 1$. Note that $\boldsymbol{\Delta}_0 \supseteq \boldsymbol{\Delta}_1 \supseteq \dots$ and $\Omega_{real} \in \boldsymbol{\Delta}_k$ for any $k \geqslant 0$.

Checking whether the intersection of ellipsoids belongs to a given ellipsoid cannot be represented as a semidefinite programming problem [22]. Therefore, we replace the set $\boldsymbol{\Delta}_k$ with its outer ellipsoidal approximation. With this approach, it is possible to find a controller minimizing the upper bound of the considered norm of the closed-loop system for all models consistent with the available data.

## 5. OUTER APPROXIMATION OF THE SET $\boldsymbol{\Delta}_k$

Summing inequalities (4.2) and (4.5) with some nonnegative multipliers $\mu \geqslant 0$ and $\tau_0 \geqslant 0, \dots, \tau_{k-1} \geqslant 0$, we establish that all matrices $\Omega$ consistent with the a priori information and current data by the time instant $k$ satisfy the inequality

$$\widehat{F}_{\mu,\tau_0^{k-1}}(\Omega) := (\Omega \quad I)\Psi(\mu, \tau_0^{k-1})(\Omega \quad I)^{\mathrm{T}} \preceq 0, \quad k \geqslant 1, \tag{5.1}$$

where

$$\Psi(\mu, \tau_0^{k-1}) = \begin{pmatrix} \mu I + \sum\limits_{t=0}^{k-1} \tau_t(\widehat{\varphi}_t \widehat{\varphi}_t^{\mathrm{T}} - d^2 I) & \star \\ -\mu\Omega_* - \sum\limits_{t=0}^{k-1} \tau_t \widehat{x}_{t+1} \widehat{\varphi}_t^{\mathrm{T}} & \mu(\Omega_*\Omega_*^{\mathrm{T}} - \rho^2 I) + \sum\limits_{t=0}^{k-1} \tau_t \widehat{x}_{t+1}\widehat{x}_{t+1}^{\mathrm{T}} - M_k \end{pmatrix}, \tag{5.2}$$

$$\tau_0^{k-1} = (\tau_0, \dots, \tau_{k-1}), \quad M_k := \sum_{t=0}^{k-1} \tau_t (I + B_w B_w^{\mathrm{T}}) d^2.$$

We define $\widehat{\boldsymbol{\Delta}}_k(\mu, \tau_0^{k-1}) = \left\{\Omega \in \mathrm{R}^{n_x \times (n_x + n_u)} : \widehat{F}_{\mu,\tau_0^{k-1}}(\Omega) \preceq 0\right\}$ as the set of all matrices $\Omega$ satisfying inequality (5.1) for fixed $\mu \geqslant 0$ and $\tau_0^{k-1} \geqslant 0$. Thus, $\boldsymbol{\Delta}_k \subseteq \widehat{\boldsymbol{\Delta}}_k(\mu, \tau_0^{k-1})$ for any $\mu \geqslant 0$, $\tau_0^{k-1} \geqslant 0$ and $\Omega_{real} \in \widehat{\boldsymbol{\Delta}}_k(\mu, \tau_0^{k-1})$. This set, with certain values of $\mu \geqslant 0$ and $\tau_0^{k-1} \geqslant 0$, will serve as an outer approximation of the set $\boldsymbol{\Delta}_k$.

Let us find boundedness conditions for the set $\widehat{\boldsymbol{\Delta}}_k(\mu, \tau_0^{k-1})$. We denote by $\Psi_{ij}(\mu, \tau_0^{k-1})$, $i, j = 1, 2$, the corresponding blocks of the matrix (5.2), whose arguments will sometimes be omitted.

**Lemma 5.1.** *If*

$$\Psi_{11} = \mu I + \sum_{t=0}^{k-1} \tau_t(\widehat{\varphi}_t \widehat{\varphi}_t^{\mathrm{T}} - d^2 I) \succ 0, \tag{5.3}$$

*then the set $\widehat{\boldsymbol{\Delta}}_k(\mu, \tau_0^{k-1})$ is a bounded matrix ellipsoid given by the inequality*

$$(\Omega + \Psi_{12}^{\mathrm{T}} \Psi_{11}^{-1}) \Psi_{11} (\Omega + \Psi_{12}^{\mathrm{T}} \Psi_{11}^{-1})^{\mathrm{T}} \preceq \Gamma_k, \tag{5.4}$$

*where $\Gamma_k = \Psi_{12}^{\mathrm{T}} \Psi_{11}^{-1} \Psi_{12} - \Psi_{22} \succeq 0$.*

*Remark 5.1.* Condition (5.3) means that based on the a priori information, the "energy" of the measured signal on the entire interval must exceed the total "energy" of the disturbance and measurement noise. If this condition fails, the set $\widehat{\boldsymbol{\Delta}}_k(\mu, \tau_0^{k-1})$ will be unbounded; see [19, Lemma 2.1].

## 6. ROBUST CONTROLLER BASED ON CURRENT DATA

To design a robust controller with a gain matrix tuned from current data, first it is necessary to solve the following auxiliary problem: determine the matrix $\Theta$ for which the upper bound of the generalized $H_\infty$ norm of the uncertain closed-loop stable system (3.1) will be minimal for all $\Omega \in \widehat{\mathbf{\Delta}}_k(\mu, \tau_0^{k-1})$ with some $\mu \geqslant 0$ and $\tau_0^{k-1} \geqslant 0$. Generally speaking, a particular Lyapunov function may exist for each model with the matrix $\Omega$; according to Lemma 3.1, it characterizes the bound of the norm considered. However, conditions for the existence of a common Lyapunov function for all $\Omega \in \widehat{\mathbf{\Delta}}_k(\mu, \tau_0^{k-1})$ will be obtained below. This introduces some conservatism but allows deriving necessary and sufficient conditions for the existence of such a function and finding the corresponding gain matrices. Note that a similar approach was used in robust control when determining the so-called quadratic stability and performing stabilization under uncertainty (for example, see [23]).

**Definition 6.1.** A controller $u_t = \Theta x_t$ is called the guaranteed generalized $H_\infty$-controller with a level $\gamma$ for the uncertain system (3.1) based on a priori information and current data by a time instant $k$ if there exists a function $V_k(x) = x^{\mathrm{T}} Y_k x$, $0 \prec Y_k \prec \gamma^2 R^{-1}$, and numbers $\mu \geqslant 0$ and $\tau_0^{k-1} \geqslant 0$ such that inequality (3.3) holds for all $\Omega \in \widehat{\mathbf{\Delta}}_k(\mu, \tau_0^{k-1})$.

According to Lemma 3.1 and Remark 3.2, the closed-loop system with the guaranteed generalized $H_\infty$-controller with a level $\gamma$ is asymptotically stable and $\|H\|_{g\infty} < \gamma$ for all models consistent with the a priori information and current data by a time instant $k$.

**Theorem 6.1.** *A controller $u_t = \Theta x_t$ is the guaranteed generalized $H_\infty$-controller with a level $\gamma$ for the uncertain system* (3.1) *based on current data by a time instant $k$ if and only if $\Theta = Q_k P_k^{-1}$ and the following LMI is solvable for $P_k = P_k^{\mathrm{T}} \succ R$, $Q_k$, $\mu \geqslant 0$, and $\tau_0^{k-1} \geqslant 0$:*

$$\begin{pmatrix} -P_k & \star & \star & \star \\ \begin{pmatrix} P_k \\ Q_k \end{pmatrix} & -\Psi_{11} & \star & \star \\ 0 & -\Psi_{21} & -\Psi_{22} - P_k + B_w B_w^{\mathrm{T}} & \star \\ (C\ D)\begin{pmatrix} P_k \\ Q_k \end{pmatrix} & 0 & 0 & -\gamma^2 I \end{pmatrix} \prec 0, \tag{6.1}$$

*where $\Psi_{ij}$, $i,j = 1,2$, denote the corresponding blocks of the matrix $\Psi(\mu, \tau_0^{k-1})$* (5.2).

*Remark 6.1.* Letting $\mu = 0$ in inequality (6.1) eliminates the a priori information from the control design; with $\tau_0^{k-1} = 0$, the current data will not be used.

*Remark 6.2.* If the matrix $\Psi(\mu, \tau_0^{k-1})$ consists of experimental data obtained on the interval $[0, k-1]$ with the open-loop uncertain system under experimenter's controls, then Theorem 6.1 remains valid. Note that in this case, with $\mu = 0$ and $\tau_0 = \cdots = \tau_{k-1} = 1$, the corresponding set $\widehat{\mathbf{\Delta}}_k(\mu, \tau_0^{k-1})$ coincides with the set of all matrices consistent with the obtained data, as established in [6, 7, 16]. This means that the approach considered here yields less conservative results than in the mentioned works.

The minimum value of $\gamma > 0$ for which inequality (6.1) is solvable will be called the guaranteed generalized $H_\infty$ norm of the uncertain system based on data by a time instant $k$. This value will be denoted by $\gamma_k^*$ whereas the corresponding gain matrix by $\Theta_k$. Since inequality (6.1) for the time instant $k$ with $\tau_{k-1} = 0$ turns into inequality (6.1) for the time instant $k-1$, the guaranteed generalized $H_\infty$ norms form a nonincreasing sequence as current data are received:

$$\gamma_0^* \geqslant \gamma_1^* \geqslant \cdots . \tag{6.2}$$

Let us proceed to the robust control design. As the gain matrix at the initial time instant, we take the matrix obtained based on only the a priori information, i.e., $\Theta_0 = Q_0 P_0^{-1}$, where $P_0$ and $Q_0$ are the solution of inequality (6.1) with $\tau_0^{k-1} = 0$ and $\gamma = \gamma_0^*$. For $t = 1, \dots, N-1$, the matrices $\Theta_t$ are computed as in Theorem 6.1 for the minimum value $\gamma = \gamma_t^*$. For $t \geqslant N$, we set $\Theta_t = \Theta_{N-1}$.

**Theorem 6.2.** *For the uncertain system* (2.1) *under the controller* $u_t = \Theta_t x_t$, *where* $\Theta_t$, $t = 0, \dots, N-1$, *are computed as in Theorem 6.1 for the minimum value of* $\gamma > 0$ *and* $\Theta_t = \Theta_{N-1}$ *for* $t \geqslant N$, *the following condition holds after the parameter tuning procedure:*

$$\sum_{t=N-1}^{\infty} |z_t|^2 < \gamma_{N-1}^{*2} \left( x_{N-1}^{\mathrm{T}} R^{-1} x_{N-1} + \sum_{t=N-1}^{\infty} |w_t|^2 \right) \quad \forall\, w_t. \tag{6.3}$$

Due to (6.2) and (6.3), in the current data-based control design, it is necessary to trade off between the quality of the closed-loop system obtained after the tuning procedure and the duration of the tuning interval (i.e., the costs): the larger $N$ is, the smaller $\gamma_{N-1}^*$ will be, and vice versa.

In the case of a sufficiently large uncertainty radius (i.e., rough a priori information), inequality (6.1) with $\tau_0^{k-1} = 0$ may have no solutions. This means the absence of a common stabilizing controller for all models from the initial set $\boldsymbol{\Delta}_0$. In this case, a random vector is chosen as the initial control action. If at the next step, still no common controller exists for the new set of models $\widehat{\boldsymbol{\Delta}}_1(\mu, \tau_0) \subseteq \boldsymbol{\Delta}_0$, then a random control action is chosen again, and this process continues until a time instant $t_*$ when inequality (6.1) will have a solution. The guaranteed estimate of the control performance (see Theorem 6.2) remains valid in this case if the initial time is taken as $t = t_*$.

## 7. AN ILLUSTRATIVE EXAMPLE

Consider a model of vibrations of a three-story building under seismic load on its foundation. Figure 1 shows the general scheme of this process. The building stories are represented as material points, serially connected to each other and to the foundation by linear elastic and dissipative elements. Vibrations generated by the seismic load (foundation movement) occur in the horizontal plane. Assume that the building is homogeneous, i.e., the masses of the material points, as well as

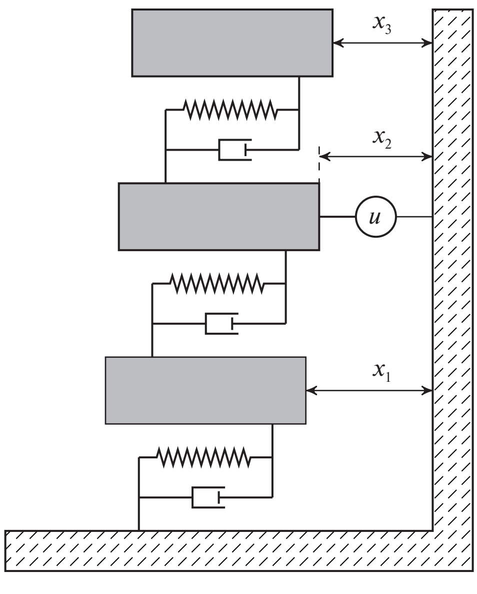


**Fig. 1.** The scheme of a building as a multimass elastic system.

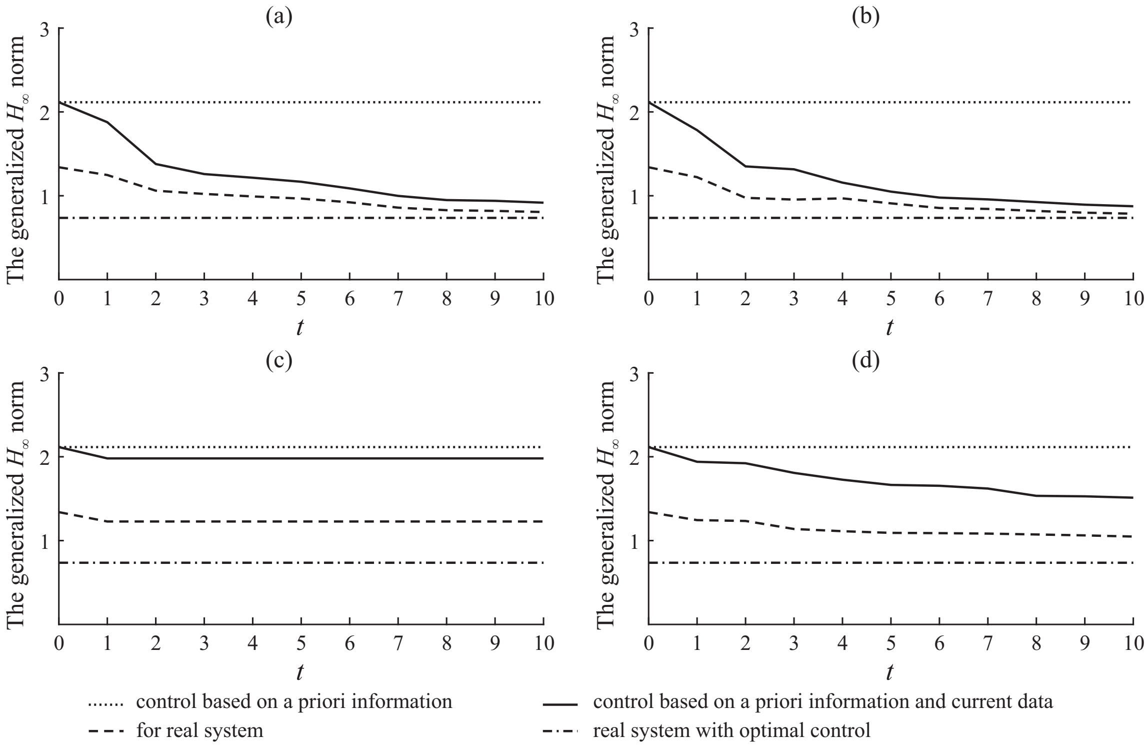


**Fig. 2.** The evolution of the guaranteed generalized $H_\infty$ norms using different information with $\varepsilon_\xi = 0.01$ in the following cases: (a) $\varepsilon_w = 0.01$ and $\varepsilon_0 = 2$, (b) $\varepsilon_w = 0.01$ and $\varepsilon_0 = 5$, (c) $\varepsilon_w = 0.1$ and $\varepsilon_0 = 2$, and (d) $\varepsilon_w = 0.1$ and $\varepsilon_0 = 5$.

the elasticity and damping coefficients of the elastic and dissipative elements, are the same. Under this assumption, the dynamics of the structure under consideration (in dimensionless variables and parameters) are described by the system of differential equations

$$\begin{aligned}\ddot{x}_1 &= -\beta(2\dot{x}_1 - \dot{x}_2) - (2x_1 - x_2) + w,\\ \ddot{x}_2 &= -\beta(2\dot{x}_2 - \dot{x}_1 - \dot{x}_3) - (2x_2 - x_1 - x_3) + u + w,\\ \ddot{x}_3 &= -\beta(\dot{x}_3 - \dot{x}_2) - (x_3 - x_2) + w,\\ z &= -x_1 - \beta\dot{x}_1 + \alpha u\end{aligned} \tag{7.1}$$

with the following notation: $x_i, i = 1, 2, 3$, are the coordinates of the material points relative to the moving foundation; $u$ is the damping force generated by the active vibration protection system installed in the building; $w$ is the acceleration of the foundation, specified up to the sign; $z$ is the scalar performance output defining the maximum force counteracting the displacement of the elastic system relative to the base; finally, $\beta$ is the damping parameter. The vibration isolation problem is to find a controller minimizing $z$ in some sense.

Let us pass from the continuous-time system to a discrete one with a discretization step of $h = 0.5$. We set the following numerical values of the parameters: $\alpha = 0.1$, $\beta = 0.1$, and $R = 0.01I$. The center of the matrix sphere $\mathbf{\Delta}^{(a)}$ consists of the discrete model matrices of system (7.1) with $\beta = 0.2$ and inequality (4.1) holds for $\rho = 0.07$. The state measurement noise, disturbance, and initial state were chosen randomly on the spheres of radii $\varepsilon_\xi$, $\varepsilon_w$, and $\varepsilon_0$, respectively; the control measurements were noiseless, i.e., $\varepsilon_\eta = 0$. In total, the uncertain system (7.1) in canonical form contains 49 unknown parameters.

Figure 2 presents the following evolutions in the case $\varepsilon_\xi = 0.01$: the guaranteed generalized $H_\infty$ norm $\gamma_t^*$ of the uncertain system (7.1) under the controller $u_t = \Theta_t x_t$ based on the a priori informa-

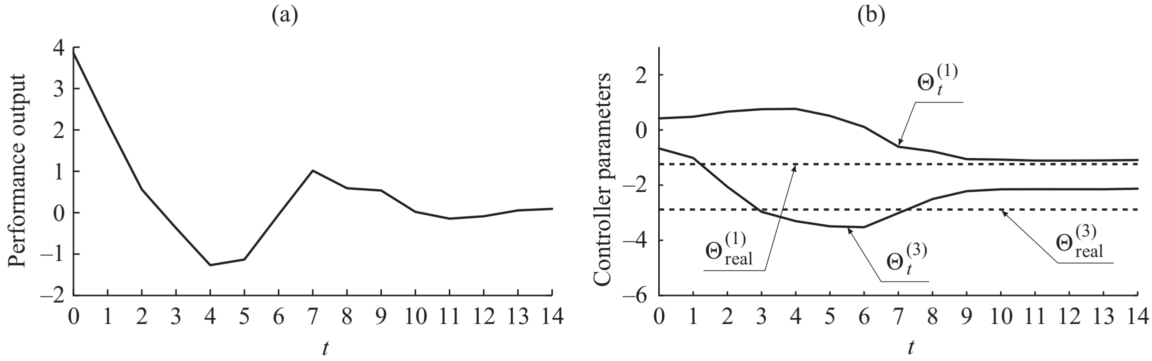


**Fig. 3.** The evolution of (a) the performance output and (b) two tunable parameters for the real system under $u_t = \Theta_t x_t$.

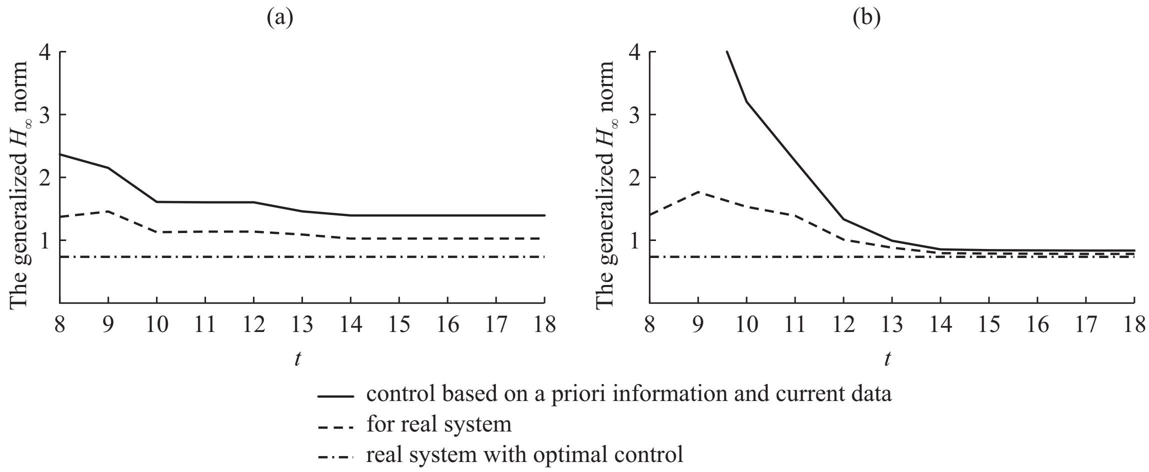


**Fig. 4.** The evolution of the guaranteed generalized $H_\infty$ norm for $\varepsilon_\xi = 0.01$ and $\varepsilon_w = 0.1$ in the case of no common robust controller based on a priori information: (a) $\varepsilon_0 = 3$ and (b) $\varepsilon_0 = 5$.

tion and current data (solid curve); the generalized $H_\infty$ norm $\|H\|_{g\infty}(\Omega_{real}, \Theta)$ for the closed-loop system consisting of the real system and the time-invariant state-feedback controller with $\Theta = \Theta_t$ (the dotted curve); the guaranteed generalized $H_\infty$ norm of the uncertain system (7.1) with the robust controller $u_t = \Theta_0 x_t$ obtained based on only the a priori information (the dashed line); finally, the optimal $H_\infty$ norm under the optimal controller for the real closed-loop system (the dash-and-dot line). Direct comparison of cases (a) and (b) or (c) and (d) in this figure shows that an increase in the initial state "increases" the block $\Psi_{11}$ and, accordingly, decreases the matrix ellipsoid $\widehat{\mathbf{\Delta}}_k(\mu, \tau_0^{k-1})$, ultimately accelerating the convergence of the generalized $H_\infty$ norm to the minimum value. Comparing cases (a) and (b) with (c) and (d), we observe that a higher value of the disturbance amplitude slows down the convergence rate of the index, up to a possible stop. The reason lies in expanding the set of models consistent with the obtained data. Next, Fig. 3 illustrates typical trajectories of the performance output and two of the six control parameters based on the a priori information and current data for the real system.

According to Fig. 4, even when the a priori information with $\rho = 0.2$ is inaccurate and the guaranteed generalized $H_\infty$ norm of the uncertain system (7.1) with the robust controller $u_t = \Theta_0 x_t$ obtained based on only the a priori information cannot be determined, the current information turns out to be sufficient for inequality (6.1) to become solvable, by the eighth step for $\varepsilon_0 = 3$ and by the tenth step for $\varepsilon_0 = 5$, and the controller is quickly retuned.

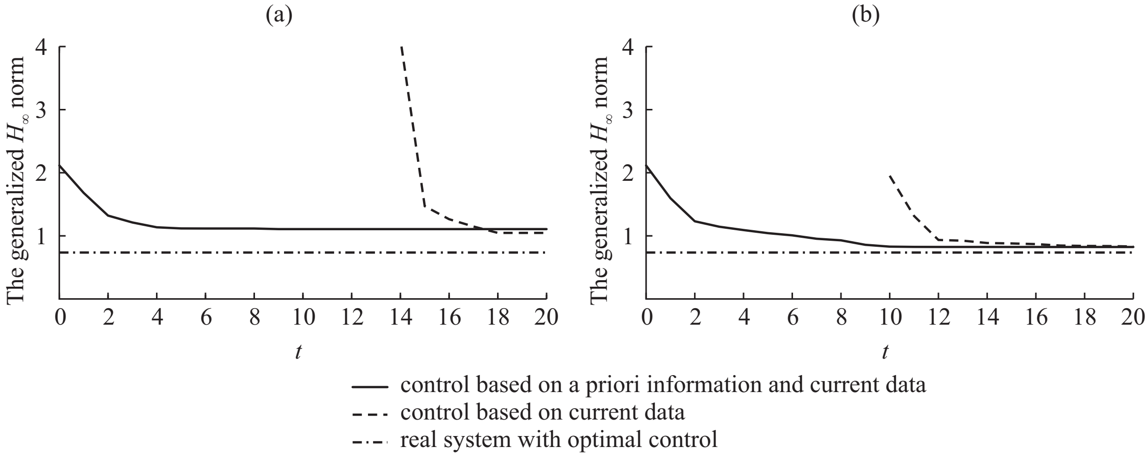


**Fig. 5.** The evolution of the guaranteed generalized $H_\infty$ norms using both the a priori information and current data and only current data for $\varepsilon_\xi = 0.01$ and $\varepsilon_w = 0.1$ : (a) $\varepsilon_0 = 3$ and (b) $\varepsilon_0 = 5$.

For the sake of comparison, Fig. 5 shows the evolution of the guaranteed generalized $H_\infty$ norms when using both the a priori information and current data (the solid curve) and only the current data (the dotted curve). Clearly, the a priori information plays a positive role in the control design: the controller based on only the current data "turns on" merely at steps 14 or 10, whereas the controller based on the combined information provides an acceptable system performance level from the first step.

## 8. CONCLUSIONS

This paper has developed a new control design method for an unknown dynamic system based on a priori information and noisy current data obtained during the real-time operation of the system. At each step, the gain matrix is found by solving linear matrix inequalities to minimize the upper bound of the generalized $H_\infty$ norm of the closed-loop system for all models consistent with the data obtained by this step. As has been proven, the sequence of the guaranteed values of the corresponding norm obtained in this way is monotonically nonincreasing and, when the tuning procedure stops, the resulting controller ensures a certain performance level of the closed-loop system. A distinctive feature of this approach is that the persistent excitation condition is not required for system identification. Furthermore, unlike adaptive control (where the main issue is to establish the convergence of the tunable parameters to their true values, and the performance of the resulting system cannot be assessed), it is possible to track the value of the optimized functional in real time. The numerical simulation of the active protection of buildings against seismic loads has demonstrated the effectiveness of the proposed approach.

## FUNDING

This work was supported by the Russian Science Foundation, project no. 24-11-20023.

*APPENDIX*

**Proof of Lemma 4.1.** For matrices $\Omega$ consistent with the data $\widehat{x}_t, \widehat{x}_{t+1}$ and $\widehat{u}_t$, equality (4.4) holds for some disturbances and noises satisfying the constraints (2.3). It follows that

$$(\widehat{x}_{t+1} - \Omega\widehat{\varphi}_t)(\widehat{x}_{t+1} - \Omega\widehat{\varphi}_t)^{\mathrm{T}}$$
$$= (I\ \Omega\ B_w) \begin{pmatrix} \xi_{t+1} \\ \zeta_t \\ w_t \end{pmatrix} \begin{pmatrix} \xi_{t+1} \\ \zeta_t \\ w_t \end{pmatrix}^{\mathrm{T}} (I\ \Omega\ B_w)^{\mathrm{T}} \preceq d^2\, (I\ \Omega\ B_w)\, (I\ \Omega\ B_w)^{\mathrm{T}}. \tag{A.1}$$

Writing this condition equivalently as

$$\Omega(\widehat{\varphi}_t\widehat{\varphi}_t^{\mathrm{T}} - d^2 I)\Omega^{\mathrm{T}} - \Omega\widehat{\varphi}_t\widehat{x}_{t+1}^{\mathrm{T}} - \widehat{x}_{t+1}\widehat{\varphi}_t^{\mathrm{T}}\Omega^{\mathrm{T}} + \widehat{x}_{t+1}\widehat{x}_{t+1}^{\mathrm{T}} - (I + B_w B_w^{\mathrm{T}})d^2 \preceq 0,$$

we obtain (4.5). In turn, if the inequality in (A.1) is valid, equality (4.4) will hold [13] for some disturbance and noises satisfying the constraints (2.3).

**Proof of Lemma 5.1.** Writing inequality (5.1) as

$$\Omega\Psi_{11}\Omega^{\mathrm{T}} + \Omega\Psi_{12} + \Psi_{12}^{\mathrm{T}}\Omega^{\mathrm{T}} + \Psi_{22} \preceq 0$$

and completing the square, we bring it to the form (5.4). Since $\Omega_{real} \in \widehat{\mathbf{\Delta}}_k(\mu, \tau_0^{k-1})$, this set is nonempty and, consequently, $\Gamma_k \succeq 0$. Multiplying (5.4) by $a^{\mathrm{T}}$ and $a$ on the left and right, respectively, and considering $\Psi_{11} > 0$, we arrive at

$$|(\Omega + \Psi_{12}^{\mathrm{T}}\Psi_{11}^{-1})^{\mathrm{T}} a|^2 \leqslant \lambda_{\min}^{-1}(\Psi_{11})\lambda_{\max}(\Gamma_k) \quad \forall\, a : |a| = 1$$

$$\Rightarrow \|(\Omega + \Psi_{12}^{\mathrm{T}}\Psi_{11}^{-1})\|^2 \leqslant \lambda_{\min}^{-1}(\Psi_{11})\lambda_{\max}(\Gamma_k),$$

i.e., the set $\widehat{\mathbf{\Delta}}_k(\mu, \tau_0^{k-1})$ is bounded.

**Proof of Theorem 6.1.** Let inequality (6.1) be solvable for some $P_k = P_k^{\mathrm{T}} \succ R$, $Q_k$, $\mu \geqslant 0$, and $\tau_0^{k-1} \geqslant 0$. We replace $Q_k = \Theta P_k$ there and apply the Schur complement lemma to the negative definite block in the first row and first column to get the equivalent inequality

$$\begin{pmatrix} \Upsilon - \Psi_{11} & \star & \star \\ -\Psi_{21} & -\Psi_{22} - P_k + B_w B_w^{\mathrm{T}} & \star \\ (C\ D)\Upsilon & 0 & (C\ D)\Upsilon(C\ D)^{\mathrm{T}} - \gamma^2 I \end{pmatrix} \prec 0,$$

$$\Upsilon = \begin{pmatrix} I \\ \Theta \end{pmatrix} P_k \begin{pmatrix} I \\ \Theta \end{pmatrix}^{\mathrm{T}}.$$

Direct substitution shows that the last inequality is true if and only if, for all nonzero $x_t^{(a)}$, $w_t^{(a)}$, and $w_t^{\Delta}$, the function $V_k(x^{(a)}) = x^{(a)\mathrm{T}} P_k x^{(a)}$ with $P_k \succ R$ satisfies the inequality

$$\Delta V_k(x_t^{(a)}) + |z_t^{(a)}|^2 - \gamma^2 |w_t^{(a)}|^2 - \begin{pmatrix} w_t^{\Delta} \\ x_t^{(a)} \end{pmatrix}^{\mathrm{T}} \Psi(\mu, \tau_0^{k-1}) \begin{pmatrix} w_t^{\Delta} \\ x_t^{(a)} \end{pmatrix} < 0; \tag{A.2}$$

here, $\Delta V_k(x_t^{(a)}) = V_k(x_{t+1}^{(a)}) - V_k(x_t^{(a)})$ denotes the increment of the function $V_k(x^{(a)})$ along the trajectory of the system

$$\begin{aligned} x_{t+1}^{(a)} &= \begin{pmatrix} I \\ \Theta \end{pmatrix}^{\mathrm{T}} w_t^{\Delta} + \begin{pmatrix} I \\ \Theta \end{pmatrix}^{\mathrm{T}} (C\ D)^{\mathrm{T}}\, w_t^{(a)}, \\ z_t^{(a)} &= B_w^{\mathrm{T}} x_t^{(a)}. \end{aligned} \tag{A.3}$$

Further, letting $w_t^{\Delta} = \Omega^{\mathrm{T}} x_t^{(a)}$ in (A.3) yields

$$\begin{aligned} x_{t+1}^{(d)} &= \begin{pmatrix} I \\ \Theta \end{pmatrix}^{\mathrm{T}} \Omega^{\mathrm{T}} x_t^{(d)} + \begin{pmatrix} I \\ \Theta \end{pmatrix}^{\mathrm{T}} (C\ D)^{\mathrm{T}}\, w_t^{(d)}, \\ z_t^{(d)} &= B_w^{\mathrm{T}} x_t^{(d)}, \end{aligned} \tag{A.4}$$

and these equations coincide with those of the dual system for the original one

$$
\begin{aligned}
x_{t+1} &= \Omega \begin{pmatrix} I \\ \Theta \end{pmatrix} x_t + B_w w_t, \\
z_t &= (C\ D) \begin{pmatrix} I \\ \Theta \end{pmatrix} x_t.
\end{aligned}
\tag{A.5}
$$

Hence, for all nonzero variables $x_t^{(d)}$ and $w_t^{(d)}$ of system (A.4), the function $V_k(x^{(d)}) = x^{(d)\mathrm{T}} P_k x^{(d)}$ with $P_k \succ R$ satisfies the inequalities

$$
\Delta V_k(x_t^{(d)}) + |z_t^{(d)}|^2 - \gamma^2 |w_t^{(d)}|^2 - x_t^{(d)\mathrm{T}} \begin{pmatrix} \Omega^{\mathrm{T}} \\ I \end{pmatrix}^{\mathrm{T}} \Psi(\mu, \tau_0^{k-1}) \begin{pmatrix} \Omega^{\mathrm{T}} \\ I \end{pmatrix} x_t^{(d)} < 0. \tag{A.6}
$$

Since we have $(\Omega\, I)\Psi(\mu, \tau_0^{k-1})(\Omega\, I)^{\mathrm{T}} \preceq 0$ for all $\Omega \in \widehat{\mathbf{\Delta}}_k(\mu, \tau_0^{k-1})$, the function $V_k(x^{(d)}) = x^{(d)\mathrm{T}} P_k x^{(d)}$ with $P_k \succ R$ satisfies the inequalities

$$
\Delta V_k(x_t^{(d)}) + |z_t^{(d)}|^2 - \gamma^2 |w_t^{(d)}|^2 < 0 \tag{A.7}
$$

for all nonzero variables $x_t^{(d)}$ and $w_t^{(d)}$ and all $\Omega \in \widehat{\mathbf{\Delta}}_k(\mu, \tau_0^{k-1})$. Due to Lemma 3.2, it follows that $u_t = \Theta x_t$ is the guaranteed generalized $H_\infty$-controller with the level $\gamma$.

Conversely, let $u_t = \Theta x_t$ be the guaranteed generalized $H_\infty$-controller with a level $\gamma$. By the definition and Lemma 3.2, for the dual system (A.4), there exist a function $V_k(x^{(d)}) = x^{(d)\mathrm{T}} P_k x^{(d)}$ with $P_k \succ R$ and numbers $\widehat{\mu} \geqslant 0$ and $\widehat{\tau}_0^{k-1} \geqslant 0$ such that inequality (A.7) holds for all $\Omega \in \widehat{\mathbf{\Delta}}_k(\widehat{\mu}, \widehat{\tau}_0^{k-1})$. We show that along the trajectory of system (A.3), the function $V_k(x^{(a)}) = x^{(a)\mathrm{T}} P_k x^{(a)}$ with $P_k \succ R$ satisfies the inequality

$$
\Delta V_k(x_t^{(a)}) + |z_t^{(a)}|^2 - \gamma^2 |w_t^{(a)}|^2 < 0 \tag{A.8}
$$

for all nonzero $x_t^{(a)}$, $w_t^{(a)}$, and $w_t^{\Delta}$ such that

$$
\begin{pmatrix} w_t^{\Delta} \\ x_t^{(a)} \end{pmatrix}^{\mathrm{T}} \Psi(\widehat{\mu}, \widehat{\tau}_0^{k-1}) \begin{pmatrix} w_t^{\Delta} \\ x_t^{(a)} \end{pmatrix} \leqslant 0. \tag{A.9}
$$

Indeed, we choose $x_t^{(a)} = x_t^{(d)}$ and $w_t^{(a)} = w_t^{(d)}$ and, for each $w_t^{\Delta}$ satisfying (A.9), define $\Omega$ as the solution of the linear matrix equation $x_t^{(a)\mathrm{T}} \Omega = w_t^{\Delta\mathrm{T}}$, containing one equation for each column of the matrix $\Omega$. In this case, from (A.9) it follows that $x_t^{(a)\mathrm{T}}(\Omega\, I)\Psi(\widehat{\mu}, \widehat{\tau}_0^{k-1})(\Omega\, I)^{\mathrm{T}} x_t^{(a)} \leqslant 0$, i.e., $\Omega \in \widehat{\mathbf{\Delta}}_k(\widehat{\mu}, \widehat{\tau}_0^{k-1})$. Thus, for such $\Omega$, equations (A.3) coincide with equations (A.4), and inequality (A.8) coincides with inequality (A.7).

By the losslessness of the $S$-procedure with one constraint, inequality (A.8) under the constraint (A.9) is equivalent to the inequality

$$
\Delta V_k(x_t^{(a)}) + |z_t^{(a)}|^2 - \gamma^2 |w_t^{(a)}|^2 - \nu \begin{pmatrix} w_t^{\Delta} \\ x_t^{(a)} \end{pmatrix}^{\mathrm{T}} \Psi(\widehat{\mu}, \widehat{\tau}_0^{k-1}) \begin{pmatrix} w_t^{\Delta} \\ x_t^{(a)} \end{pmatrix} < 0
$$

for some $\nu \geqslant 0$. Note that by assumption, $\Omega_{real}$ lies strictly inside $\widehat{\mathbf{\Delta}}_k(\widehat{\mu}, \widehat{\tau}_0^{k-1})$, so inequality (A.9) is strict for $w_t^{\Delta} = \Omega_{real}^{\mathrm{T}} x_t^{(a)}$. Since $\Psi(\mu, \tau_0^{k-1})$ linearly depends on $\mu$ and $\tau_0^{k-1}$, denoting $\nu\widehat{\mu} = \mu$ and $\nu\widehat{\tau}_0^{k-1} = \tau_0^{k-1}$, we obtain inequality (A.2), which is equivalent to the LMI (6.1) for $Q_k = \Theta P_k$.

**Proof of Theorem 6.2.** Similar to the proof of Theorem 6.1 for $k = N-1$, we obtain that along the trajectory of system (A.4), the function $V_{N-1}(x_t^{(d)}) = x_t^{(d)\mathrm{T}} P_{N-1} x_t^{(d)}$ with $P_{N-1} > R$ satisfies inequality (A.6) for all $\Omega \in \widehat{\boldsymbol{\Delta}}_{N-1}(\mu, \tau_0^{N-1})$. Consequently, by Lemmas 3.1 and 3.2 and Remark 3.1, along the trajectory of system (2.1) with $u_t = \Theta_{N-1} x_t$, the function $V_{N-1}(x_t) = x_t^{\mathrm{T}} Y_{N-1} x_t$, where $Y_{N-1} = \gamma_{N-1}^{*2} P_{N-1}^{-1}$, satisfies the inequalities

$$\triangle V_{N-1}(x_t) + |z_t|^2 - \gamma_{N-1}^{*2} |w_t|^2 < 0$$

for all $\Omega \in \widehat{\boldsymbol{\Delta}}_{N-1}(\mu, \tau_0^{N-1})$ and $t \geqslant N-1$. Summing these inequalities starting from $t = N-1$ and considering the relations $\lim_{t\to\infty} V_{N-1}(x_t) = 0$ and $Y_{N-1} < \gamma_{N-1}^{*2} R^{-1}$, we finally arrive at (6.3).

## REFERENCES


1. Polyak, B.T. and Shcherbakov, P.S., *Robastnaya ustoichivost' i upravlenie* (Robust Stability and Control), Moscow: Nauka, 2002.

2. Petersen, I.R. and Tempo, R., Robust Control of Uncertain Systems: Classical Results and Recent Developments, *Automatica*, 2014, vol. 50, no. 5, pp. 1315–1335.

3. Markovsky, I., Huang, L., and Dörfler, F., Data-Driven Control Based on the Behavioral Approach: From Theory to Applications in Power Systems, *IEEE Control Syst. Mag.*, 2023, vol. 43, no. 5, pp. 28–68.

4. van Waarde, H.J., Eising, J., Camlibel, M.K., and Trentelman, H.L., The Informativity Approach: To Data-Driven Analysis and Control, *IEEE Control Syst. Mag.*, 2023, vol. 43, no. 6, pp. 32–66.

5. Willems, J.C., Rapisarda, P., Markovsky, I., and De Moor, B., A Note on Persistency of Excitation, *Syst. Control Lett.*, 2005, vol. 54, pp. 325–329.

6. De Persis, C. and Tesi, P., Formulas for Data-Driven Control: Stabilization, Optimality and Robustness, *IEEE Trans. Automat. Control*, 2020, vol. 65, no. 3, pp. 909–924.

7. van Waarde, H.J., Eising, J., Trentelman, H.L., and Camlibel, M.K., Data Informativity: A New Perspective on Data-Driven Analysis and Control, *IEEE Trans. Automat. Control*, 2020, vol. 65, no. 11, pp. 4753–4768.

8. Berberich, J., Koch, A., Scherer, C.W., and Allgower, F., Robust Data-Driven State-Feedback Design, *Proc. Amer. Control Conf.*, 2020, pp. 1532–1538.

9. Bisoffi, A., De Persis, C., and Tesi, P., Trade-Offs in Learning Controllers from Noisy Data, *Syst. Control Lett.*, 2021, vol. 154, art. no. 104985.

10. van Waarde, H.J., Camlibel, M.K., and Mesbahi, M., From Noisy Data to Feedback Controllers: Nonconservative Design via a Matrix S-Lemma, *IEEE Trans. Automat. Control*, 2022, vol. 67, no. 1, pp. 162–175.

11. Bisoffi, A., De Persis, C., and Tesi, P., Data-Driven Control via Petersen's Lemma, *Automatica*, 2022, vol. 145, art. no. 110537.

12. Nortmann, B. and Mylvaganam, T., Direct Data-Driven Control of LTV Systems, *IEEE Trans. Automat. Control*, 2023, vol. 68, pp. 4888–4895.

13. Bisoffi, A., De Persis, C., and Monshizadeh, N., Controller Synthesis for Input-State Data with Measurement Errors, *IEEE Control Syst. Lett.*, 2024, vol. 8, pp. 1571–1576.

14. Berberich, J., Scherer, C.W., and Allgower, F., Combining Prior Knowledge and Data for Robust Controller Design, *IEEE Trans. Automat. Control*, 2023, vol. 68, no. 8, pp. 4618–4633.

15. Kogan, M.M. and Stepanov, A.V., Design of Suboptimal Robust Controllers Based on A Priori and Experimental Data, *Autom. Remote Control*, 2023, vol. 84, no. 8, pp. 918–932.

16. Li, L., De Persis, C., Tesi, P., and Monshizadeh, N., Data-Based Transfer Stabilization in Linear Systems, *IEEE Trans. Automat. Control*, 2024, vol. 69, pp. 1866–1873.

17. Kogan, M.M. and Stepanov, A.V., Design of Generalized $H_\infty$-Suboptimal Controllers Based on Experimental and A Priori Data, *Autom. Remote Control*, 2024, vol. 85, no. 1, pp. 1–14.

18. Niknejad, N. and Modares, H., Physics-Informed Data-Driven Safe and Optimal Control Design, *IEEE Control Syst. Lett.*, 2024, vol. 8, pp. 285–290.

19. Kogan, M.M. and Stepanov, A.V., How to Improve Robust Control of a Linear Time-Varying System by Using Experimental Data, *Autom. Remote Control*, 2024, vol. 85, no. 6, pp. 636–654.

20. Kogan, M.M. and Stepanov, A.V., Robust $H_\infty$ Control with Transients for LTV Systems Based on Prior Knowledge and Data, *Automatica*, 2025, vol. 173, art. no. 112083.

21. Balandin, D.V., Biryukov, R.S., and Kogan, M.M., Pareto Suboptimal $H_\infty$ Controls with Transients, *Proc. Eur. Control Conf.*, 2021, Rotterdam, pp. 542–547.

22. Boyd, S., El Ghaoui, L., Feron, E., and Balakrishnan, V., *Linear Matrix Inequalities in System and Control Theory*, Philadelphia: SIAM, 1994.

23. Petersen, I.R. and McFarlane, D.C., Optimal Guaranteed Cost Control and Filtering for Uncertain Linear Systems, *IEEE Trans. Automat. Control*, 1994, vol. 30, no. 9, pp. 1971–1977.

24. Yakubovich, V.A., $S$-Procedure in Nonlinear Control Theory, *Vestn. Leningrad. Univ. Mat.*, 1977, no. 4, pp. 73–93.

*This paper was recommended for publication by P.S. Shcherbakov, a member of the Editorial Board*